

# FAT Review

Solve:  $[0, 360)$

$$1) 2\sin^2(x) - 3\sin(x) + 1 = 0$$

$$2\sin^2(x) - 2\sin(x) - \sin(x) + 1 = 0$$

$$2\sin(x)(\sin(x)-1) + -1(\sin(x)-1) = 0$$

$$(2\sin(x)-1)(\sin(x)-1) = 0$$

$$2\sin(x)-1 = 0$$

$$\sin(x) = \frac{1}{2}$$

$$x = 30^\circ, 150^\circ$$

$$\sin(x)-1 = 0$$

$$\sin(x) = 1$$

$$x = 90^\circ$$

$$2) 2\cos(x) - 1 = 0$$

$$\cos(x) = \frac{1}{2}$$

$$x = 60^\circ, 300^\circ$$

$$3) \cos(x) - 2\cos(x)\sin(x) = 0$$

$$\cos(x)(1-2\sin(x)) = 0$$

$$\cos(x) = 0$$

$$x = 90^\circ, 270^\circ$$

$$1-2\sin(x) = 0$$

$$\frac{1}{2} = \sin(x)$$

$$x = 30^\circ, 150^\circ$$

$$4) 4\sin^2(x) - 3 = 0$$

$$\sin(x) = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

$$\sin(x) = \frac{\sqrt{3}}{2}$$

$$x = 60^\circ, 120^\circ$$

$$\sin(x) = -\frac{\sqrt{3}}{2}$$

$$x = 240^\circ, 300^\circ$$

Verify. You can only manipulate one side.

$$5) \sin(\theta)\cot(\theta) = \cos(\theta)$$

$$\cancel{\sin(\theta)} \cdot \frac{\cos(\theta)}{\cancel{\sin(\theta)}} =$$

$$\cos(\theta) = \checkmark$$

$$6) \frac{1}{\sec^2(\theta)} + \frac{1}{\csc^2(\theta)} - 1 = 0$$

$$\cos^2(\theta) + \sin^2(\theta) - 1 =$$

$$1 - 1 =$$

$$0 = \checkmark$$

$$7) \cos^2(x) + \tan^2(x)\cos^2(x) = 1$$

$$\cos^2(x)(1 + \tan^2(x)) =$$

$$\cos^2(x) \cdot \sec^2(x) =$$

$$\cos^2(x) \cdot \frac{1}{\cos^2(x)} =$$

$$1 = \checkmark$$

$$8) \frac{\cos(x)}{\cot^2(x)} = \sin(x)\tan(x)$$

$$\frac{\cos(x)}{\frac{1}{\sin^2(x)}} =$$

$$\frac{\sin^2(x)}{\cos(x)} =$$

$$\sin(x)\tan(x) = \checkmark$$

Simplify

$$9) \frac{\cos^2(x)}{1 - \cos^2(x)}$$

$$= \frac{\cos^2(x)}{\sin^2(x)}$$

$$= \cot^2(x)$$

$$10) \frac{\sin^2(x) \cot(x)}{\cos(x)}$$

$$= \frac{\sin^2(x) \frac{\cos(x)}{\sin(x)}}{\cos(x)}$$

$$= \frac{\sin(x) \cancel{\cos(x)}}{\cos(x)}$$

$$= \sin(x)$$

$$11) \frac{\sec^2(x) - 1}{\sin^2(x)}$$

$$= \frac{\tan^2(x)}{\sin^2(x)}$$

$$= \frac{\frac{\sin^2(x)}{\cos^2(x)}}{\sin^2(x)}$$

$$= \frac{1}{\cos^2(x)}$$

$$= \sec^2(x)$$

$$12) \tan^2(x) (\csc^2(x) - 1)$$

$$= \tan^2(x) \cdot \cot^2(x)$$

$$= \frac{\sin^2(x)}{\cos^2(x)} \cdot \frac{\cos^2(x)}{\sin^2(x)}$$

$$= 1$$

$$13) \cot^2(x) \sin^2(x) + \tan^2(x) \cos^2(x)$$

$$= \frac{\cos^2(x)}{\sin^2(x)} \cdot \sin^2(x) + \frac{\sin^2(x)}{\cos^2(x)} \cdot \cos^2(x)$$

$$= (\cos^2(x) + \sin^2(x))$$

$$= 1$$

Verify

$$14) \frac{1 + \tan^2(x)}{\tan^2(x)} = \csc^2(x)$$

$$\frac{\sec^2(x)}{\tan^2(x)} =$$

$$\frac{\frac{1}{\cos^2(x)}}{\frac{\sin^2(x)}{\cos^2(x)}} =$$

$$\frac{1}{\sin^2(x)} = \csc^2(x) = \checkmark$$

$$15) \frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)} = \frac{\csc(x)}{\cos(x)}$$

$$\frac{\sin^2(x) + \cos^2(x)}{\sin(x) \cos(x)} =$$

$$\frac{1}{\sin(x) \cos(x)} =$$

$$\frac{1}{\sin(x)} \cdot \frac{1}{\cos(x)} =$$

$$\csc(x) \cdot \frac{1}{\cos(x)} =$$

$$\frac{\csc(x)}{\cos(x)} = \checkmark$$

$$16) 2 \sec^2(x) - 2 \sec^2(x) \sin^2(x) - \sin^2(x) - \cos^2(x) = 1$$

$$2 \sec^2(x) (1 - \sin^2(x)) - \sin^2(x) - \cos^2(x) =$$

$$\frac{2}{\cos^2(x)} \cdot \cos^2(x) - (\sin^2(x) + \cos^2(x)) =$$

$$2 - 1 = 1 = \checkmark$$

Solve:  $[0, 360)$

17)  $\cos^2(x) \sin(x) - \cos^2(x) = 0$

$$\cos^2(x) (\sin(x) - 1) = 0$$

$$\cos^2(x) = 0$$

$$\cos(x) = 0$$

$$x = 90^\circ, 270^\circ$$

$$\sin(x) - 1 = 0$$

$$\sin(x) = 1$$

$$x = 90^\circ$$

18)  $\tan^2(x) - 3 = 0$

$$\tan(x) = \sqrt{3}$$

$$x = 60^\circ, 240^\circ$$

19)  $\sin^2(x) = \cos(x) - 1$

$$1 - \cos^2(x) = \cos(x) - 1$$

$$0 = \cos^2(x) + \cos(x) - 2$$

$$0 = (\cos(x) + 2)(\cos(x) - 1)$$

$$\cos(x) + 2 = 0$$

$$\cos(x) = -2$$

$$x = \text{N/A, none}$$

$$\cos(x) - 1 = 0$$

$$\cos(x) = 1$$

$$x = 0^\circ$$

20)  $3 \tan^2(x) - 1 = 0$

$$\tan(x) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3}$$

$$\tan(x) = \frac{\sqrt{3}}{3}$$

$$x = 30^\circ, 210^\circ$$