Precalculus Law of Cosine I

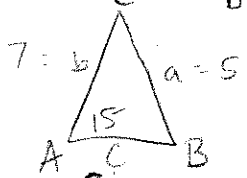
Determine if you would use Law of Sine or Law of Cosine. If you are using Law of Sine, determine the number of solutions.

① $a = 5$ $b = 7$ $c = 8$ SSS

Law of Cosine

② $A = 15°$ $a = 5$ $b = 7$ ASS

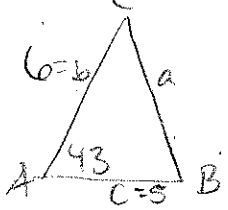
$b\sin(A) = 7\sin(15) = 1.81 < 5 < 7$



Two Solutions

Law of sine

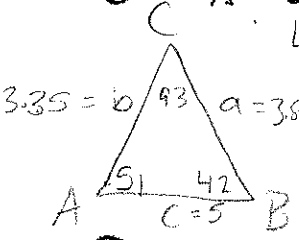
③ $A = 43°$ $b = 6$ $c = 5$ SAS



Law of Cosine

④ $A = 51°$ $B = 42°$ $c = 5$ ASA

Law of Sine (1 sol)



$\frac{\sin(93)}{5} = \frac{\sin(42)}{b}$

$b = \frac{5\sin(42)}{\sin(93)}$

$b = 3.35$

$\frac{\sin(93)}{5} = \frac{\sin(51)}{a}$

$a = \frac{5\sin(51)}{\sin(93)}$

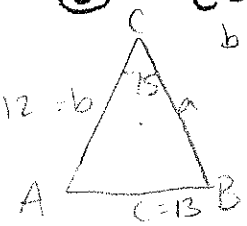
$= 3.89$

⑤ $a = 10$ $b = 5$ $c = 5\sqrt{3}$ SSS

Law of Cosine

⑥ $C = 75°$ $b = 12$ $c = 13$

$b\sin(C) = 12\sin(75) = 11.59 < 13 > 12$

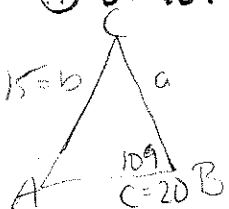


Law of Sine

1 solution

ASS

⑦ $B = 109°$ $b = 15$ $c = 20$ ASS



Law of Sine

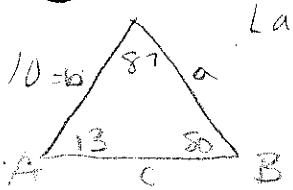
No Solution

⑧ $A = 13°$ $C = 87°$ $b = 10$

Law of Sine

1 solution

ASA

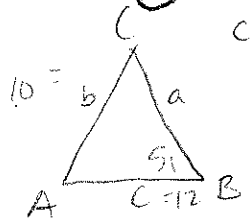


⑨ $a = 15$ $b = 7$ $c = 12$ SSS

Law of Cosine

⑩ $B = 51°$ $c = 12$ $b = 10$ Law of sines

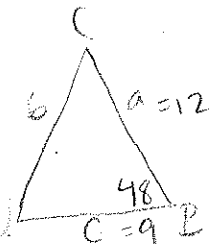
$C\sin(B) = 12\sin(51) = 9.33 < 10 < 12$



2 Solutions

ASS

⑪ $B = 48°$ $c = 9$ $a = 12$ SAS



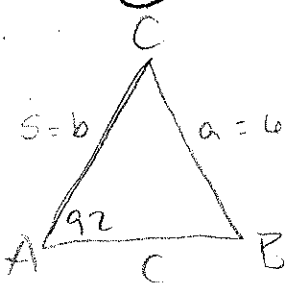
Law of Cosine

⑫ $A = 92°$ $a = 6$ $b = 5$

Law of Sines

ASS

1 solution



Precalculus Law of Cosine 2

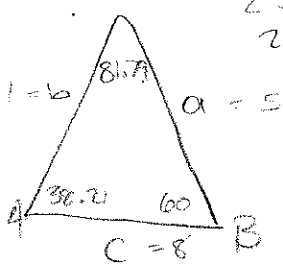
$a^2 = b^2 + c^2 - 2bc\cos(A)$

$b^2 = a^2 + c^2 - 2ac\cos(B)$

$c^2 = a^2 + b^2 - 2ab\cos(C)$

Solve using Law of Cosine.

a = 5 b = 7 c = 8



$5^2 = 7^2 + 8^2 - 2(7)(8)\cos(A)$

$25 = 49 + 64 - 112\cos(A)$

$25 = 113 - 112\cos(A)$

$-88 = -112\cos(A)$

$\frac{11}{14} = \cos(A)$

$A = \cos^{-1}\left(\frac{11}{14}\right) = 38.21$

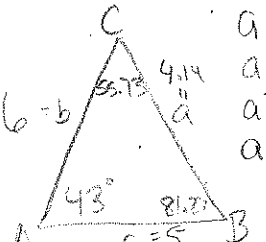
$\frac{\sin(38.21)}{5} = \frac{\sin(B)}{7}$

$B = \sin^{-1}\left(\frac{7\sin(38.21)}{5}\right)$

≈ 60

$180 - 60 - 38.21$

② A = 43° b = 6 c = 5



$a^2 = 6^2 + 5^2 - 2(5)(6)\cos(43)$

$a^2 = 36 + 25 - 60\cos(43)$

$a^2 = 61 - 60\cos(43)$

$a = \sqrt{61 - 60\cos(43)} \approx 4.14$

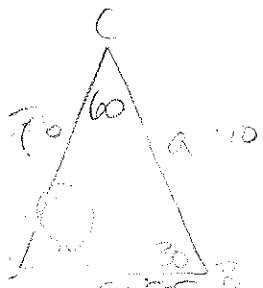
$\frac{\sin(43)}{4.14} = \frac{\sin(B)}{6}$

$B = \sin^{-1}\left(\frac{6\sin(43)}{4.14}\right)$

≈ 81.27

$180 - 43 - 81.27$

③ a = 10 b = 5 c = $5\sqrt{3}$



$(5\sqrt{3})^2 = 10^2 + 5^2 - 2(10)(5)\cos(C)$

$75 = 100 + 25 - 100\cos(C)$

$75 = 125 - 100\cos(C)$

$-50 = -100\cos(C)$

$\frac{1}{2} = \cos(C)$

$C = \cos^{-1}\left(\frac{1}{2}\right) = 60$

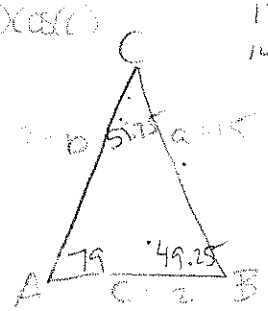
$\frac{\sin(60)}{5\sqrt{3}} = \frac{\sin(A)}{10}$

$A = \sin^{-1}\left(\frac{10\sin(60)}{5\sqrt{3}}\right)$

$= 90$

$180 - 90 - 60$

④ a = 15 b = 7 c = 12



$12^2 = 15^2 + 7^2 - 2(15)(7)\cos(C)$

$144 = 225 + 49 - 210\cos(C)$

$144 = 274 - 210\cos(C)$

$-130 = -210\cos(C)$

$\frac{13}{21} = \cos(C)$

$C = \cos^{-1}\left(\frac{13}{21}\right) \approx 51.75$

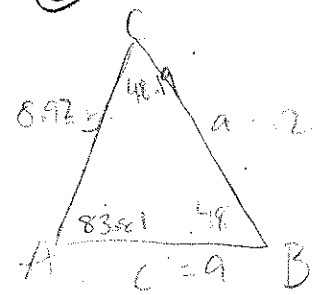
$\frac{\sin(51.75)}{12} = \frac{\sin(A)}{15}$

$A = \sin^{-1}\left(\frac{15\sin(51.75)}{12}\right)$

≈ 79

$180 - 79 - 51.75$

⑤ B = 48° c = 9 a = 12



$b^2 = 12^2 + 9^2 - 2(12)(9)\cos(48)$

$b^2 = 144 + 81 - 216\cos(48)$

$b^2 = 225 - 216\cos(48)$

$b = \sqrt{225 - 216\cos(48)}$

≈ 8.97

$\frac{\sin(48)}{8.97} = \frac{\sin(A)}{12}$

$A = \sin^{-1}\left(\frac{12\sin(48)}{8.97}\right)$

≈ 83.81

$180 - 83.81 - 48$

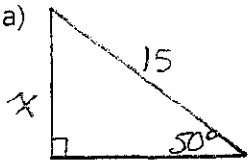
Answer Key

FAT : Unit 9 Right Triangles Review Name Key

1. Write the six Trig Ratios of a right triangle.

a) $\sin(\theta) = \frac{O}{H}$ b) $\cos(\theta) = \frac{A}{H}$ c) $\tan(\theta) = \frac{O}{A}$ d) $\csc(\theta) = \frac{H}{O} = \frac{1}{\sin(\theta)}$ e) $\sec(\theta) = \frac{H}{A} = \frac{1}{\cos(\theta)}$ f) $\cot(\theta) = \frac{A}{O} = \frac{1}{\tan(\theta)}$

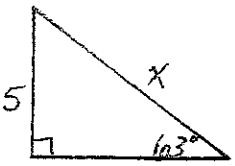
2. Find the missing parts of each right triangle.

a) 

$\sin(50) = \frac{X}{15}$

$15\sin(50) = X$

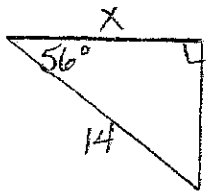
$11.49 = X$

b) 

$\sin(63) = \frac{5}{X}$

$X = \frac{5}{\sin(63)}$

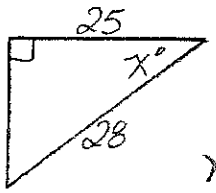
$X = 5.61$

c) 

$\cos(56) = \frac{X}{14}$

$14\cos(56) = X$

$7.83 = X$

d) 

$\cos(x) = \frac{25}{28}$

$X = \cos^{-1}\left(\frac{25}{28}\right)$

$x = 26.77°$

3. Solve each of the following triangles.

If $\sin\theta = \frac{O}{H} = \frac{3}{5}$, find $\tan\theta$

$3^2 + X^2 = 5^2$

$X^2 = 5^2 - 3^2$

$X^2 = 16$

$X = 4$

$\tan(\theta) = \frac{3}{4}$

b) If $\tan\theta = \frac{O}{A} = \frac{\sqrt{3}}{2}$, find $\csc\theta$

$(\sqrt{3})^2 + 2^2 = H^2$

$3 + 4 = H^2$

$7 = H^2$

$\sqrt{7} = H$

$\csc(\theta) = \frac{\sqrt{7}}{\sqrt{3}}$

c) If $\cot\theta = .8$, find the $\sec\theta$

$\frac{4}{5} = \frac{A}{O}$

$4^2 + 5^2 = H^2$

$16 + 25 = H^2$

$41 = H^2$

$\sqrt{41} = H$

$\sec(\theta) = \frac{\sqrt{41}}{4}$

d) If $\cos\theta = \frac{4}{1} = \frac{A}{H}$

← NOT possible. Hypothenus must be longest side

4. How many solutions? (answer the following questions)

a) What are the four types of triangles (using triangle shortcuts) that have **only** one triangle possible.

SSS, SAS, ASA, AAS

b) Given an S-S-A triangle what condition(s) create:

	Given angle $\geq 90°$	Given angle $< 90°$
• 2 possible triangles	0 sol	adj sin(angle) > opp
• 1 possible triangle	1 sol. opp > adj	adj sin(angle) < opp > adj
• No possible triangle	2 sol.	adj sin(angle) < opp < adj

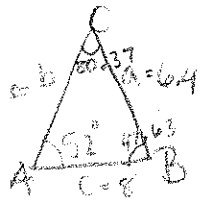
Law of Cosine, Law of Sine I

Name: _____

HW 12

Solve each of the following triangles. Show all set-ups.

1. $A = 52°$ $b = 6$ $c = 8$ SAS



$a^2 = 6^2 + 8^2 - 2(6)(8)\cos(52)$

$a^2 = 36 + 64 - 96\cos(52)$

$a^2 = 100 - 96\cos(52)$

$a = \sqrt{100 - 96\cos(52)}$

$a = 6.4$

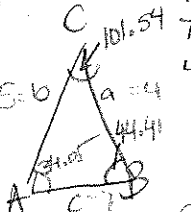
$\frac{\sin(52)}{6.4} = \frac{\sin(B)}{6}$

$B = \sin^{-1}\left(\frac{6\sin(52)}{6.4}\right)$

$B = 47.63$

$C = 80.37$

2. $a = 4$ $b = 5$ $c = 7$ SSS



$7^2 = 4^2 + 5^2 - 2(4)(5)\cos(C)$

$49 = 16 + 25 - 40\cos(C)$

$49 = 41 - 40\cos(C)$

$8 = -40\cos(C)$

$-\frac{1}{5} = \cos(C)$

$C = \cos^{-1}(-\frac{1}{5}) = 101.54$

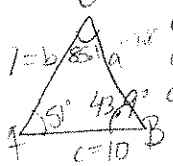
$\frac{\sin(101.54)}{7} = \frac{\sin(A)}{4}$

$A = \sin^{-1}\left(\frac{4\sin(101.54)}{7}\right)$

$A = 34.05°$

$B = 44.41°$

3. $b = 7$ $c = 10$ $A = 51°$ SAS



$a^2 = 7^2 + 10^2 - 2(7)(10)\cos(51)$

$a^2 = 49 + 100 - 140\cos(51)$

$a^2 = 149 - 140\cos(51)$

$a = \sqrt{149 - 140\cos(51)}$

$a = 7.8$

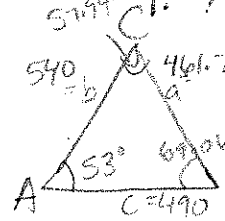
$\frac{\sin(51)}{7.8} = \frac{\sin(C)}{10}$

$C = \sin^{-1}\left(\frac{10\sin(51)}{7.8}\right)$

$C = 85.1$

$B = 43.9$

4. $A = 53°$ $b = 540$ $c = 490$ SAS



$a^2 = 540^2 + 490^2 - 2(540)(490)\cos(53)$

$a^2 = 531700 - 529200\cos(53)$

$a = \sqrt{531700 - 529200\cos(53)}$

$a = 461.76$

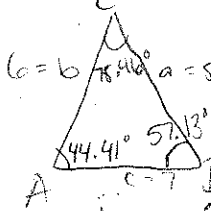
$C = 57.94°$

$\frac{\sin(53)}{461.76} = \frac{\sin(B)}{540}$

$B = \sin^{-1}\left(\frac{540\sin(53)}{461.76}\right)$

$B = 69.06$

5. $a = 5$ $b = 6$ $c = 7$ SSS



$7^2 = 5^2 + 6^2 - 2(5)(6)\cos(C)$

$49 = 61 - 60\cos(C)$

$-12 = -60\cos(C)$

$\frac{1}{5} = \cos(C)$

$C = \cos^{-1}(\frac{1}{5}) = 78.46°$

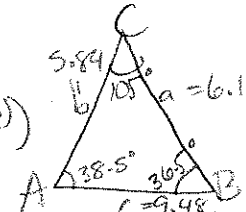
$\frac{\sin(78.46)}{7} = \frac{\sin(A)}{5}$

$A = \sin^{-1}\left(\frac{5\sin(78.46)}{7}\right)$

$A = 44.41°$

$B = 57.13°$

6. $C = 105°$ $a = 6.11$ $b = 5.84$ SAS



$c^2 = 6.11^2 + 5.84^2 - 2(6.11)(5.84)\cos(105)$

$c^2 = 71.44 - 71.36\cos(105)$

$c = \sqrt{71.44 - 71.36\cos(105)}$

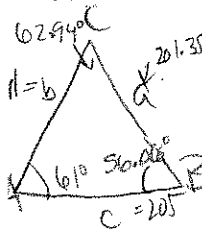
$c = 9.48$

$B =$

$\frac{\sin(105)}{9.48} = \frac{\sin(A)}{6.11}$

$A = \sin^{-1}\left(\frac{6.11\sin(105)}{9.48}\right) = 38.5°$

7. $A = 61°$ $b = 191$ $c = 205$ SAS



$a^2 = 191^2 + 205^2 - 2(191)(205)\cos(61)$

$a^2 = 78506 - 78310\cos(61)$

$a = \sqrt{78506 - 78310\cos(61)}$

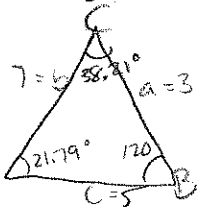
$a = 201.35$

$C = 62.94°$

$\frac{\sin(61)}{201.35} = \frac{\sin(B)}{191}$

$B = \sin^{-1}\left(\frac{191\sin(61)}{201.35}\right) = 56.06$

8. $a = 3$ $b = 7$ $c = 5$ SSS



$5^2 = 3^2 + 7^2 - 2(3)(7)\cos(C)$

$25 = 58 - 42\cos(C)$

$-33 = -42\cos(C)$

$\frac{11}{14} = \cos(C)$

$C = \cos^{-1}(\frac{11}{14}) = 38.21°$

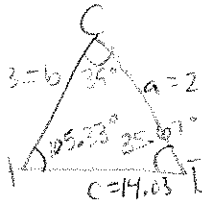
$\frac{\sin(38.21)}{5} = \frac{\sin(A)}{3}$

$A = \sin^{-1}\left(\frac{3\sin(38.21)}{5}\right)$

$A = 21.79°$

$B = 120°$

9. $b = 13$ $a = 21.5$ $C = 39°$ SAS



$c^2 = 21.5^2 + 13^2 - 2(21.5)(13)\cos(39)$

$c^2 = 631.25 - 559\cos(39)$

$c = \sqrt{631.25 - 559\cos(39)}$

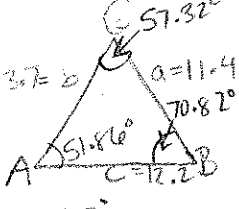
$c = 14.03$

$A = 105.33$

$\frac{\sin(39)}{14.03} = \frac{\sin(B)}{13}$

$B = \sin^{-1}\left(\frac{13\sin(39)}{14.03}\right) = 35.67$

10. $a = 11.4$ $b = 13.7$ $c = 12.2$ SSS



$12.2^2 = 11.4^2 + 13.7^2 - 2(11.4)(13.7)\cos(C)$

$148.84 = 317.65 - 312.36\cos(C)$

$-168.81 = -312.36\cos(C)$

$0.54 = \cos(C)$

$C = \cos^{-1}(0.54) = 57.32°$

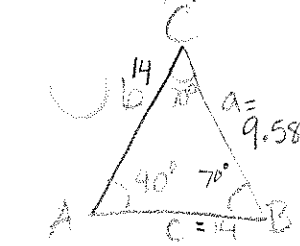
$B = 70.82°$

$\frac{\sin(57.32)}{12.2} = \frac{\sin(A)}{11.4}$

$A = \sin^{-1}\left(\frac{11.4\sin(57.32)}{12.2}\right)$

$A = 51.86°$

11. $A = 40°$ $C = 70°$ $c = 14$ AAS



$B = 70°$

$\frac{\sin(70)}{14} = \frac{\sin(C)}{b}$

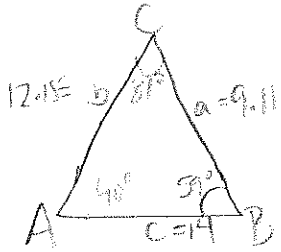
$b = 14$

$\frac{\sin(70)}{14} = \frac{\sin(40)}{a}$

$a = \frac{14\sin(40)}{\sin(70)} = 9.58$

12. $A = 40°$ $B = 59°$ $c = 14$ ASA

$C = 81°$



$\frac{\sin(81)}{14} = \frac{\sin(40)}{a}$

$a = \frac{14\sin(40)}{\sin(81)}$

$a = 9.11$

$\frac{\sin(81)}{14} = \frac{\sin(59)}{b}$

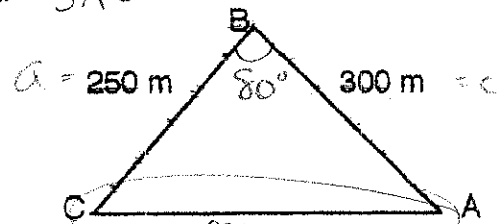
$b = \frac{14\sin(59)}{\sin(81)}$

$b = 12.15$

Name Key

Trig Analysis Law of Sine/ Law of Cosine Word Problems

1. To approximate the length of a marsh, a surveyor walks 300 meters from point A to point B, then turns 80° and walks 250 meters to point C. Approximate the length of the marsh. SAS



$b^2 = a^2 + c^2 - 2ac\cos(B)$

$b^2 = 250^2 + 300^2 - 2(250)(300)\cos(80)$

$b^2 = 62500 + 90000 - 150000\cos(80)$

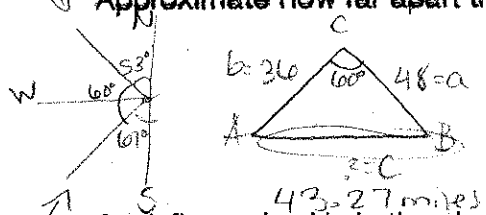
$b^2 = 152500 - 150000\cos(80)$

$b = \sqrt{152500 - 150000\cos(80)}$

$b \approx 355.6$ meters

12·3=36

2. Two ships leave a port at 9 am. One travels at a bearing of 53° west of north at 12 miles per hour and the other travels bearing 67° west of south at 16 miles per hour. Approximate how far apart they are at noon that day. SAS



$c^2 = a^2 + b^2 - 2ab\cos(C)$

$c^2 = 48^2 + 36^2 - 2(48)(36)\cos(60)$

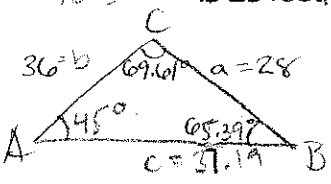
$c^2 = 2304 + 1296 - 3456\cos(60)$

$c = \sqrt{3600 - 3456\cos(60)}$

$c \approx 43.27$ miles

16·3=48

3. A flower bed is in the shape of an obtuse triangle. One angle is 45°, the side opposite is 28 feet, and another side is 36 feet. Find the remaining angles and side. ASS



$b\sin(A) = 36\sin(45) = 25.46 < 28 < 36$ → 2 Solutions

$\frac{\sin(45)}{28} = \frac{\sin(B)}{36}$ → $\angle B = 65.39°$

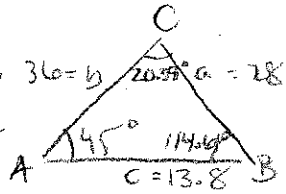
$180 - 45 - 65.39$ → $\angle C = 69.61°$

$\frac{\sin(45)}{28} = \frac{\sin(69.61)}{c}$ → $c = 37.19$

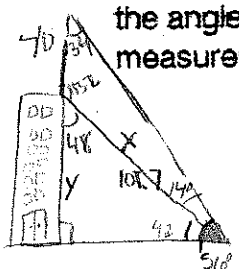
$180 - 65.39$ → $\angle B = 114.61°$

$180 - 45 - 114.61$ → $\angle C = 20.39°$

$\frac{\sin(45)}{28} = \frac{\sin(20.39)}{c}$ → $c = 13.8$



4. A 40 foot television antenna stands on top of a building. From a point on the ground, the angles of elevation of the top and bottom of the antenna, respectively have measurements of 56° and 42°. How tall is the building?



$\frac{\sin(14)}{44} = \frac{\sin(34)}{x}$

$x = \frac{44\sin(34)}{\sin(14)}$

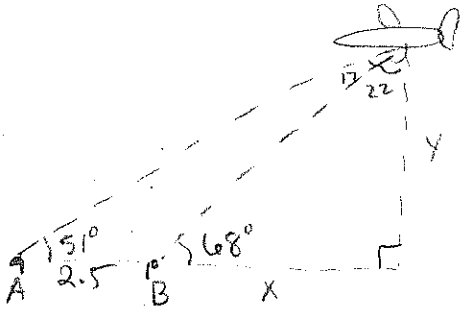
$x = 101.7$

$\sin(42) = \frac{y}{101.7}$

$101.7\sin(42) = y$

$68.05 \text{ feet} = y$

5. The angles of elevation to an airplane from two points A and B on level ground are 51° and 68°, respectively. The points A and B are 2.5 miles apart, and the airplane is east of both points in the same vertical plane. Find the altitude of the airplane.



$\tan(51) = \frac{y}{x+2.5}$ → $(x+2.5)\tan(51) = y$

$\tan(68) = \frac{y}{x}$ → $x\tan(68) = y$

$x\tan(68) = x\tan(51) + 2.5\tan(51)$

$x\tan(68) - x\tan(51) = 2.5\tan(51)$

$x(\tan(68) - \tan(51)) = 2.5\tan(51)$

$x = \frac{2.5\tan(51)}{\tan(68) - \tan(51)}$

$x = 2.49$

$2.49\tan(68) = y$

$6.16 = y$

Altitude: 6.16 miles