FAT: Higher Degree Polynomial Review

Name Key
Period ______ Date ______

State the **number** of complex roots of each equation and then actually **find the roots**.

1. $x^2 + 4 = y$

$\frac{P}{Q} = \frac{4}{1} : \frac{1,2,4}{1}$ $x = (\pm) 1, 2, 4$

Deg	2
(+)	0
(-)	0
(i)	2

$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(4)}}{2(1)}$

$x = \frac{\pm\sqrt{-16}}{2}$

$x = \frac{\pm 4i}{2}$ $\boxed{x = \pm 2i}$

2. $f(x) = x^3 + 2x^2 - 15x$

$x(x^2 + 2x - 15)$

$\frac{P}{Q} = \frac{15}{1} : \frac{1,3,5,15}{1}$ $x = (\pm) 1, 3, 5, 15$

Deg	2
(+)	1
(-)	1
(i)	0

$x^2 + 2x - 15 = (x+5)(x-3)$

$\boxed{\text{Roots: } 0, 3, -5}$

Write the polynomial equation of least degree for each set of roots given in factored form, then multiply out into standard form.

3. 4, ½

$(x-4)(x-\frac{1}{2})$

$= x^2 - \frac{1}{2}x - 4x + 2$

$= x^2 - 4\frac{1}{2}x + 2$

or

$= 2x^2 - 9x + 4$

4. 3, 3, 1, 1, -2

$(x-3)(x-3)(x-1)(x-1)(x+2)$

$(x^2 - 6x + 9)(x^2 - 2x + 1)(x+2)$

$(x^2 - 6x + 9)(x^3 - 3x + 2)$

$= x^5 - 6x^4 + 6x^3 + 20x^2 - 39x + 18$

5. -1, $\pm 3i$

$(x+1)(x+3i)(x-3i)$

$(x+1)(x^2+9)$

$= x^3 + 9x + x^2 + 9$

$= x^3 + x^2 + 9x + 9$

6. Divide using long division.

$(3x^2 + 4x - 12) \div (x - 5)$

$$
\begin{array}{r}
3x + 19 + \frac{83}{x-5} \\
x-5 \overline{)\,3x^2 + 4x - 12} \\
3x(x-5) \quad 3x^2 - 15x \quad \downarrow \\
\hline
19x - 12 \\
19(x-5) \quad 19x - 95 \\
\hline
83
\end{array}
$$

7. Divide using synthetic division.

$(x^4 - 3x + 1) \div (x - 1)$

$$
\begin{array}{r|rrrrr}
1 & 1 & 0 & 0 & -3 & 1 \\
+ & \downarrow & 1 & 1 & 1 & -2 \\
\hline
 & 1 & 1 & 1 & -2 & \boxed{-1}
\end{array}
$$

$x^3 + x^2 + x - 2 + \frac{-1}{x-1}$

Use the Factor Theorem to determine if the divisor is a factor of the polynomial. Use the Remainder Theorem to evaluate the function at the given value.

8. $(2x^3 - 3x^2 - 10x + 3) \div (x - 3)$

$$
\begin{array}{r|rrrr}
3 & 2 & -3 & -10 & 3 \\
+ & \downarrow & 6 & 9 & -3 \\
\hline
 & 2 & 3 & -1 & \boxed{0}
\end{array}
$$

Yes a factor $r = 0$

$f(3) = 2(3)^3 - 3(3)^2 - 10(3) + 3$

$= 2(27) - 3(9) - 30 + 3$

$= 54 - 27 - 30 + 3$

$= 0$

9. $(10x^3 - x^2 + 8x + 29) \div (5x + 2)$

$$
\begin{array}{r|rrrr}
-2/5 & 10 & -1 & 8 & 29 \\
+ & \downarrow & -4 & 2 & -4 \\
\hline
 & 10 & -5 & 10 & \boxed{25}
\end{array}
$$

Not a factor $r \neq 0$

$f(-\frac{2}{5}) = 10(-\frac{2}{5})^3 - (-\frac{2}{5})^2 + 8(\frac{2}{5}) + 29$

$= 10(-\frac{8}{125}) - \frac{4}{25} + \frac{16}{5} + 29$

$= -\frac{80}{125} - \frac{4}{25} + \frac{16}{5} + 29$

$= 25$

For each question, find the following:
a) Use the Rational Root Theorem to determine the possible rational roots.
b) Use Descartes Rule of Signs to find the possible number of positive, negative, and imaginary roots.
c) Use the Factor Theorem to find all the rational roots.
d) Write the polynomial in factored form.

10. $\overset{Q}{x^3} + 3x^2 - 6x - \overset{P}{8}$

$\frac{P}{Q} = \frac{8}{1} : \frac{1,2,4,8}{1}$ $\quad x = (\pm)\,1,2,4,8$

Deg	3	3
(+)	1	1
(−)	2	0
(i)	0	2

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -6 & -8 \\ + & \downarrow & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & \boxed{0} \end{array}$$

$x^2 + 2x - 8 = (x+4)(x-2)$

Roots: $-1, -4, 2$

$f(x) = (x+1)(x+4)(x-2)$

11. $x^4 - 3x^3 + 4x^2 + 8x$; with a given root of -1

$= x\,(x^3 - 3x^2 + 4x + 8)$

$\frac{P}{Q} = \frac{8}{1} : \frac{1,2,4,8}{1}$ $\quad x = (\pm)\,1,2,4,8$

Deg	3	3
(+)	2	0
(−)	1	1
(i)	0	2

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 4 & 8 \\ + & \downarrow & -1 & 4 & -8 \\ \hline & 1 & -4 & 8 & \boxed{0} \end{array}$$

$x^2 - 4x + 8 = 0$

$x = \dfrac{+4 \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$

$x = \dfrac{4 \pm \sqrt{16 - 32}}{2}$

$x = \dfrac{4 \pm \sqrt{-16}}{2}$

$x = \dfrac{4 \pm 4i}{2}$

$x = 2 \pm 2i$

Roots: $0, -1, 2+2i, 2-2i$

$f(x) = x\,(x+1)(x-(2+2i))(x-(2-2i))$

or

$= x\,(x+1)(x-2-2i)(x-2+2i)$